

Differential problems with different type solutions of mathematics education's students

Galuh Tyasing Swastika^{1,2}, Toto Nusantara¹, Subanji¹ and Santi Irawati¹

¹State University of Malang, Malang, Indonesia

²University of Nahdlatul Ulama Blitar, Blitar, Indonesia

*galuh.tyasing.1603119@students.um.ac.id, galuhtyasing@gmail.com

Abstract. Monotonous learning at the previous level resulted in a process of representing differential problem solving for students of mathematics education often limited. This can hinder their learning, especially in the lessons that require different forms of representation in their completion. This exploratory research discusses various forms of representation and reasons for their use in the differential resolution of problems in courses of calculation. The survey was conducted in a calculation class of large private universities in East Java, Indonesia. The subjects are given differential problems and they are followed for answers that require clarification. The results showed 3 types of completion based on the characteristics of the subject's response. Subject experiences in various forms of representation, as well as conceptual understanding, influence their responses in representation representation. The average subject looks at a form of representation, which is symbolic. The findings in this study can increase the awareness of educators in order to provide more experience to students with respect to the form of representation. With the broad knowledge of representation, the learning process and the understanding of various mathematical concepts can be facilitated to students.

1. Introduction

Representations should always be developed by students, students and teachers / lecturers as this is one of the important things in the basic skills of mathematics. This is stated in NCTM (2000), which representation of process standard in Principle and Standards for School Mathematics besides problem solving, reasoning, communication, and connection. In addition, Hwang & Chen (2007) also stated that representational skills are the key to successful math problem solving. Representation can be a tool for strengthening, reasoning, and communicating an idea or cognitive thinking tool (NCTM, 2000; Pantziara, Gagatsis, & Elia, 2009; Diezmann & English, 2001). Mathematical representation acts as a way of expressing mathematical ideas and the way students understand and use their mathematical ideas. Some forms of representation such as: diagrams, graphics, and symbolic expressions, have long been part of school mathematics [1].

One branch of mathematics that can facilitate some form of representation is calculus. Calculus is necessary for advanced mathematics studies that are widely used by mathematicians, statisticians, technicians and scientists. Calculus has several important components, one of which is Differential Equations. Differential equations have relevance to other mathematics courses such as, integral, many variables, numerical methods, differential equations, mathematical statistics and complex analysis. Therefore, it is very important for students especially mathematics students or mathematics education



to understand the differential equation. In this paper, the researchers will be to examine the representation of student teachers in making a graphic representation differential.

2. Literature review

The meaning of representation expressed by educational experts (Goldin & Shteingold, 2001; Kalathil & Sherin, 2000; Pape & Tchoshanov, 2001) refers to an external form to illustrate the mathematical ideas constructed in one's mind. Representation refers to both process and outcome and also gives meaning to capturing a relationship or a mathematical concept in some form, as well as in its own form [1]. With representation, students are expected to express math ideas during learning. The three things students must master in terms of representation are a) able to create, and use representations to organize, record, and communicate ideas; b) selecting, using, and translating various representations to solve problems; c) and use representations to model and interpret a phenomenon [1].

Goldin & Kaput (1996) distinguish representations that occur through two stages: internally and externally. Internal representation refers to mental configurations that allow individuals, such as learners to understand or solve problems. External representation is the result of the embodiment to describe what the student is doing (words, graphs, images and equations). Bruner (in Post, 1988) share the representation includes enaktif (enactive), iconic (iconic) and symbolic (symbolic). (1) Enactive representation is a sensorimotor representation formed by action or movement. (2) The iconic representation is a representation embodied in the form of visual images, drawings or diagrams depicting concrete activities or concrete situations found at the enactive stage. (3) Symbolic representation relates to the language of math and symbols. Lesh, Landau and Hamilton (in Post, 1988) developed a B runner representation by adding "spoken symbols and" real world situations ". So there are five types of representations: real scripts, manipulative models, static pictures, spoken language, and written symbols [10]. Of the various forms of representation available, often these forms are taught separately [1]. Whereas students need to be able to link various forms of representation according to indicators of mastery of representation on NCTM, that is students can choose, use, and translate various forms of representation in solving problems.

Janvier put forward the process of translation: "the psychological process involved in going from one mode of representation to another, for example, from an equation to a graph" [10]. This means that the process of translation is a process of change from one form of representation to another form of representation. Ainsworth (1999) defines mathematical translation as "all cases when the learner must see the relation between two representations", all events / cases when students are able to see the relationship between two representations. Translation is the process of cognition and the relation of a form of representation to a different form of representation, without changing the object which are denoted (Roth & Bowen, 2001; Duval, 2006; Bossé, Adu-Gyamfi, & Chandler, 2014). Duval (1999) divides the cognitive process in translation into two types, namely processing and conversing. Processing is meant as a translation between the same form or type of representation, for example changing an algebraic form with another algebraic form that has the same value. Conversing is the translation between two different or more representational forms, such as converting a linear equation into a graph. The term translations used in this study refers to the process of conversing, ie changing between different representations in solving math problems.

Research on the process of student translations has been done before (Bossé et al., 2014; Duru & Koklu, 2011; Hwang & Chen, 2007). Bossé et al. (2014) conducts research on the student's translation activity from graphic to symbolic based on the level of students' math skills. In Duru & Koklu's (2011) study of the translation between the form of text representation and algebra, it was found that students had difficulty translating from text form to algebraic equations using symbols. Students also have difficulty translating from the symbolic form of the text due to lack of understanding in reading. Hwang & Chen (2007) mentioned that most students do not understand the importance of their connections to various forms of representation in solving math problems. Students often use only one form of representation such as the procedure the teacher teaches. When finding difficulties in solving problems, students rarely try to use different representations to solve them. One of the mathematical material that demands the mastery of various representations is the differential equation.

Kendal & Stacey (2003) mention the goal of differential end so that students can know how to formulate, calculate, and interpret the three types of derivatives, namely the rate of change, the gradient of the tangent, and the symbolization of the derivative (at one point or as a function). Students performing differential problem resolution can interpret their solutions through symbolic, numerical, graphical, and oral representation [18]. With many types of representations on the solution of differential problems, the relationship between the different representations is required for the comprehension of the differential to be obtained in full. (Kendal & Stacey, 2003; Sofronas et al., 2011). The inability to link concepts and between representations in calculus will lead to a lack of mastery of differential problems. With the lack of mastery of concepts and various forms of representation, students could be trapped in solving only procedural differentiative problems [20].

Some research results indicate an increase in students' understanding of calculus about the differential by the use of multiple representations (Berry & Nyman, 2003; García, Llinares, & Sánchez-Matamoros, 2011). This is because students are required to connect between algebra and graphical representation or between numerical and algebraic representations. But there are still many calculus students who have difficulty making graphs of different functions even though they are adept at graphing functions, describing functional characteristics, and differentiating functions (Norman & Prichard, 1994; Baker, Cooley, & Trigueros, 2000). Therefore it is very important to conduct research on the difficulties of these students Based on the data presented, in this paper will be studied semiotic component on the representation of prospective teachers in performing differentiation graph representations.

3. Methodology

The type of research used in this research is descriptive qualitative research. This research describes student representation of teacher candidate in solution of differential problem from semiotic point of view. Differential problem used in this research is given graphic image of a function ($f(x)$), then prospective teacher students are asked to sketch their different function graphs ($f'(x)$).

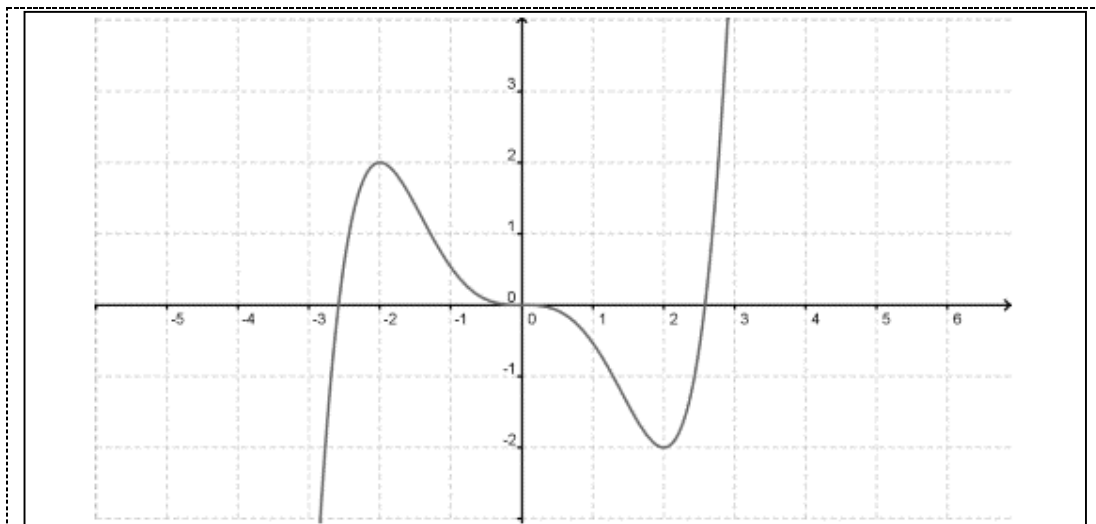


Figure 1. Differential Graphic Problem.

The methods used in this study provide online-based tests and interviews if there are unclear solutions. This research was conducted on 8 subjects of mathematics teacher candidate of a private university in East Java. Some of the selected subjects are heterogeneous and have gained prior calculus material that is in the third semester. Subject selection is also through discussions with selected university teachers. The researcher acts as the main instrument assisted by the supporting instruments of the test and confirmed by the interview.

3.1. Research Result

The following obtained the results of research on 8 subjects that are classified based on the characteristics of the subject's answer.

3.1.1. Answer Type A. Subjects in this type there is only 1 student (S1). In this type of answer, the answer is wrong. In the process of progress S1 initially do trial and error to find the target representation to get the appropriate answer. Trial and error is seen on the blur / scratch of the subject's answer in Figure 2a.

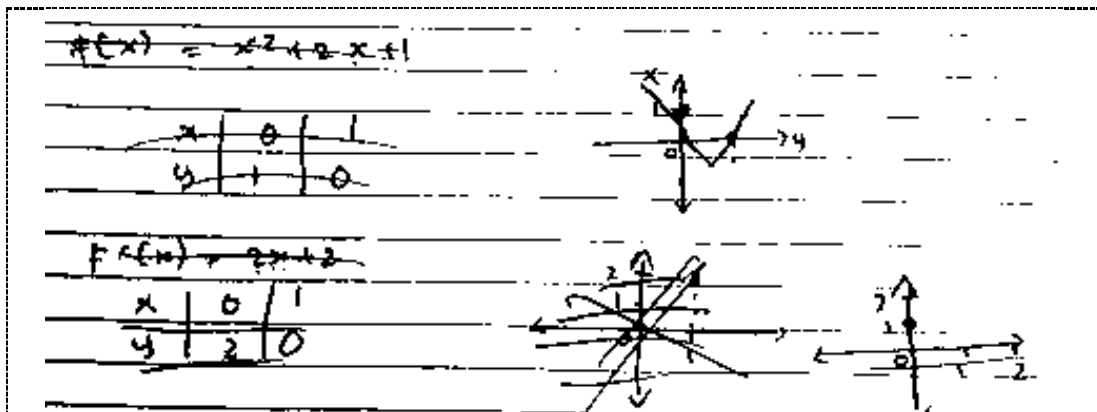


Figure 2a. Trial and Errors method.

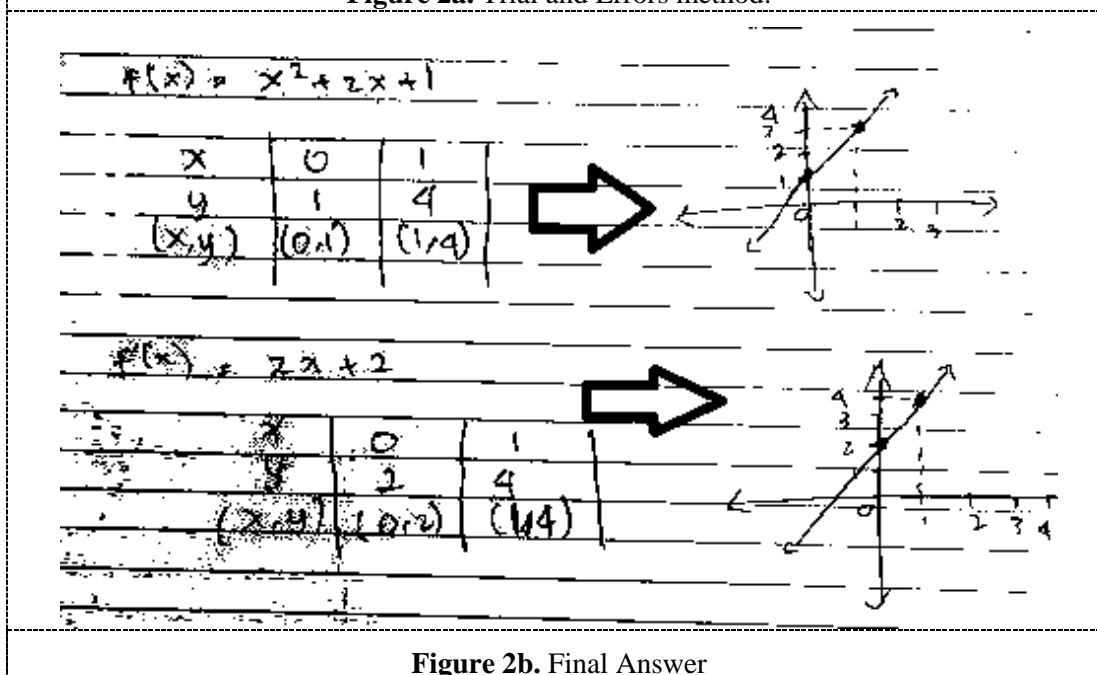


Figure 2b. Final Answer

Initially S1 tried to translate the representation source (graphic polynomial) to symbolic form. But the equations are made not in accordance with the given graph (quadratic function). Furthermore, the quadratic function that has been formed S1 is made graphic representation by registering two points on the coordinate axis. S1 also did not succeed in translating the squared function made into graphical form. The graph created is a graph for a linear function, rather than a quadratic function (see Figure 2b). However, S1 can differentiate from $f(x)$ (the quadratic function he chooses) to $f'(x)$ (linear function) symbolically appropriately. S1 can also create a graphical representation of $f'(x)$ (linear function) obtained correctly.

3.1.2. Answer Type B. Subjects in this type there are 4 students (S2, S3, S4, S5). In this type of answer, the answers are wrong and have similar characteristics. Subjects of this type respond by reading the

graph of the given function by determining the maximum and minimum points first $(-2, 2)$ the maximum value and $(2, -2)$ the minimum value (see Figures 3a and Figure 4a). Next the subject performs a calculation of the known maximum value to determine the equation of the line. But the subject determines two points (see Figures 3b and 4b in the blue box) of the maximum and minimum values for obtaining the line, which of course is not appropriate. Subjects are differentiating from the linear line function (Figure 3b and 4b red circle) they obtain, rather than the polynomial function of the given graph. The final answer of the subject is also not according to the given command that is the function of a symbolic differential rather than graphically. These subjects differed from the maximum and minimum values. This indicates they do not understand the meaning of the given question.

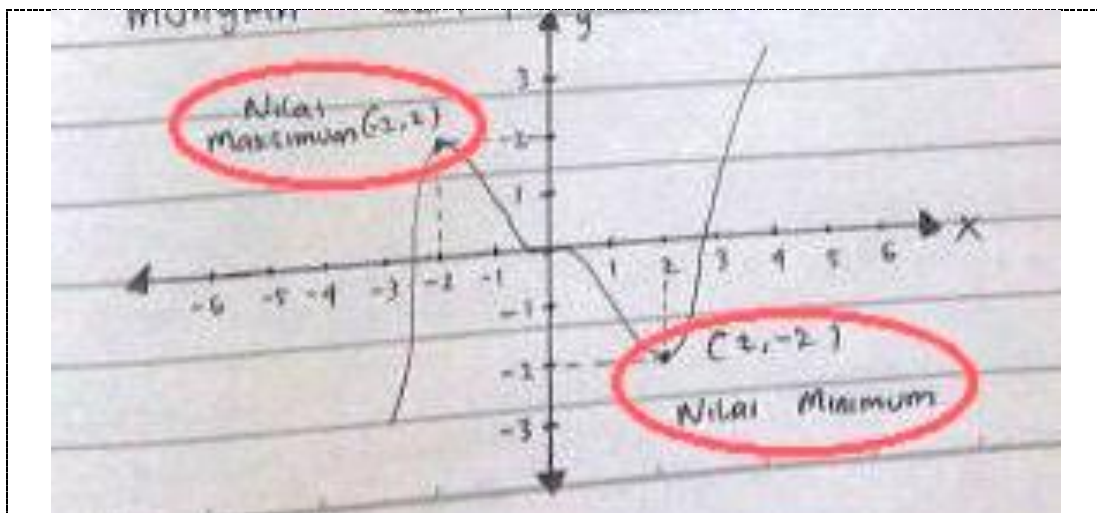


Figure 3a. Answer Type B.

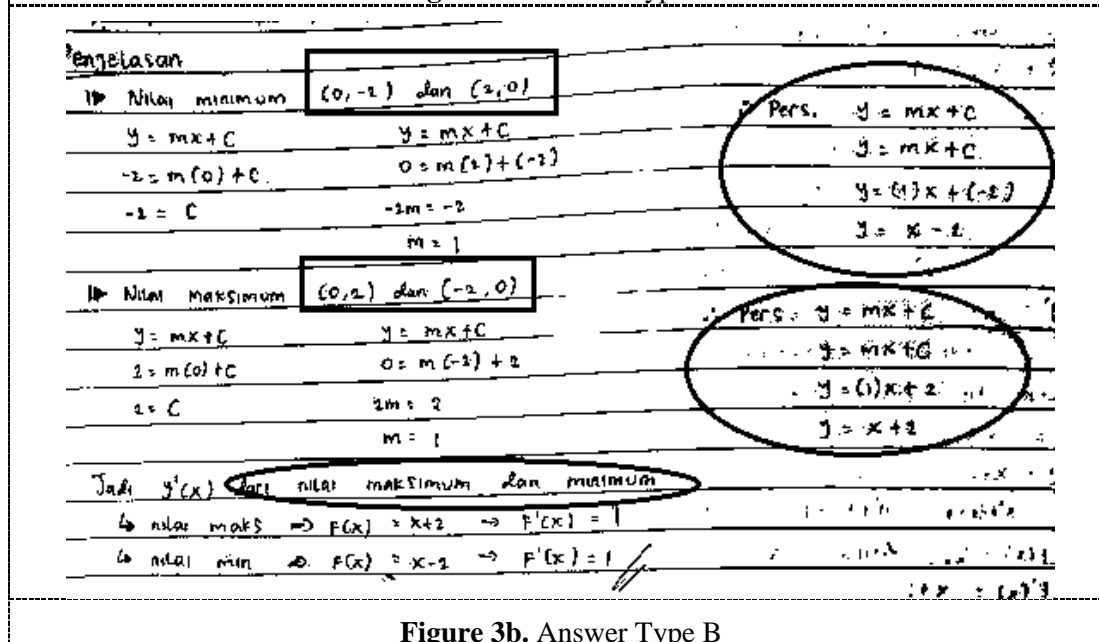


Figure 3b. Answer Type B

Similar to the one in Figure 3, one of these type B subjects (S5) gives a final answer in the form of a graph. S5 can understand the command problem to give a sketch graph (Figure 4a) rather than in symbolic form (equation). Although the answer of S5 with the other three subjects on this type is equally going in the wrong direction, ie, doing the differential to the maximum and minimum values of the linear function formed (Figure 4b).

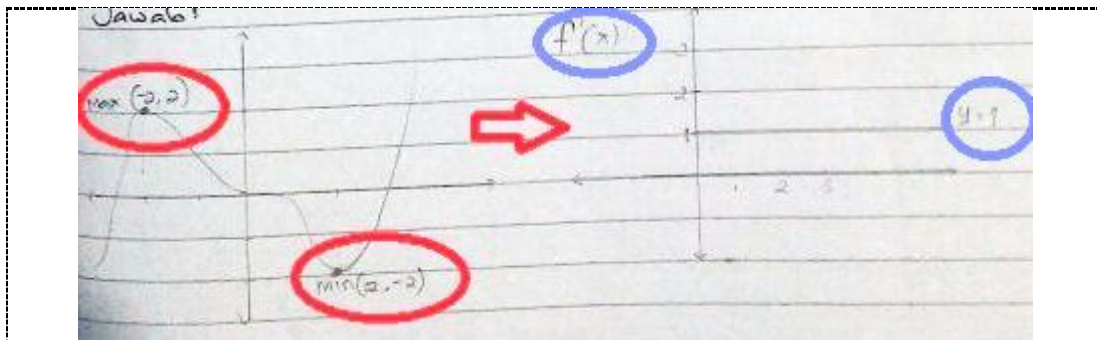


Figure 4a. Answer Type B.

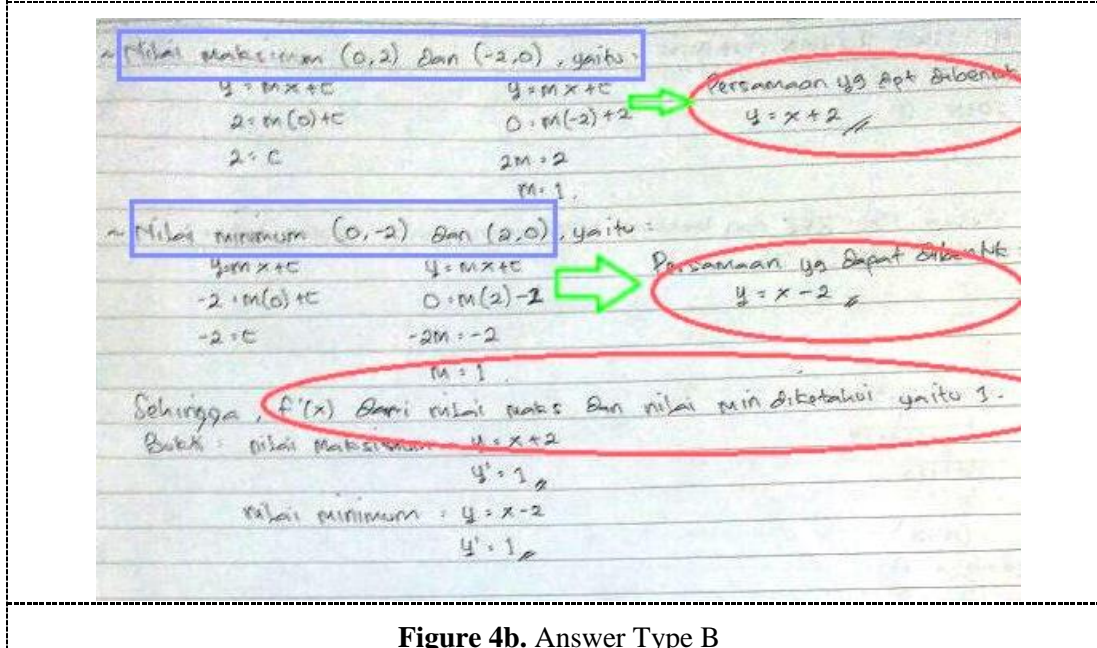
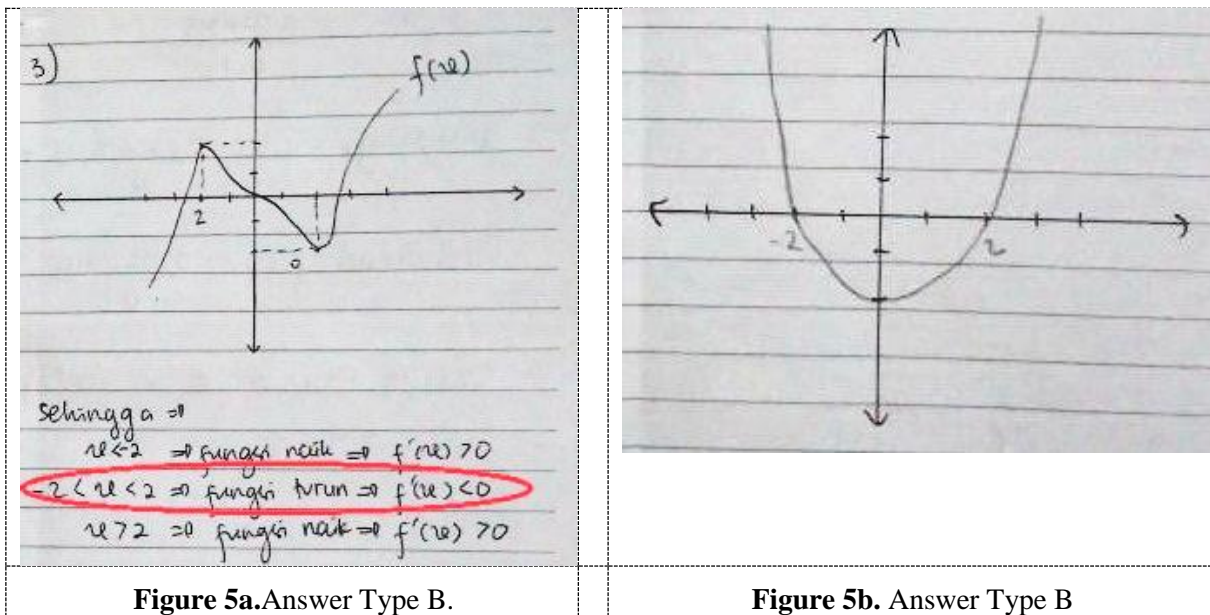


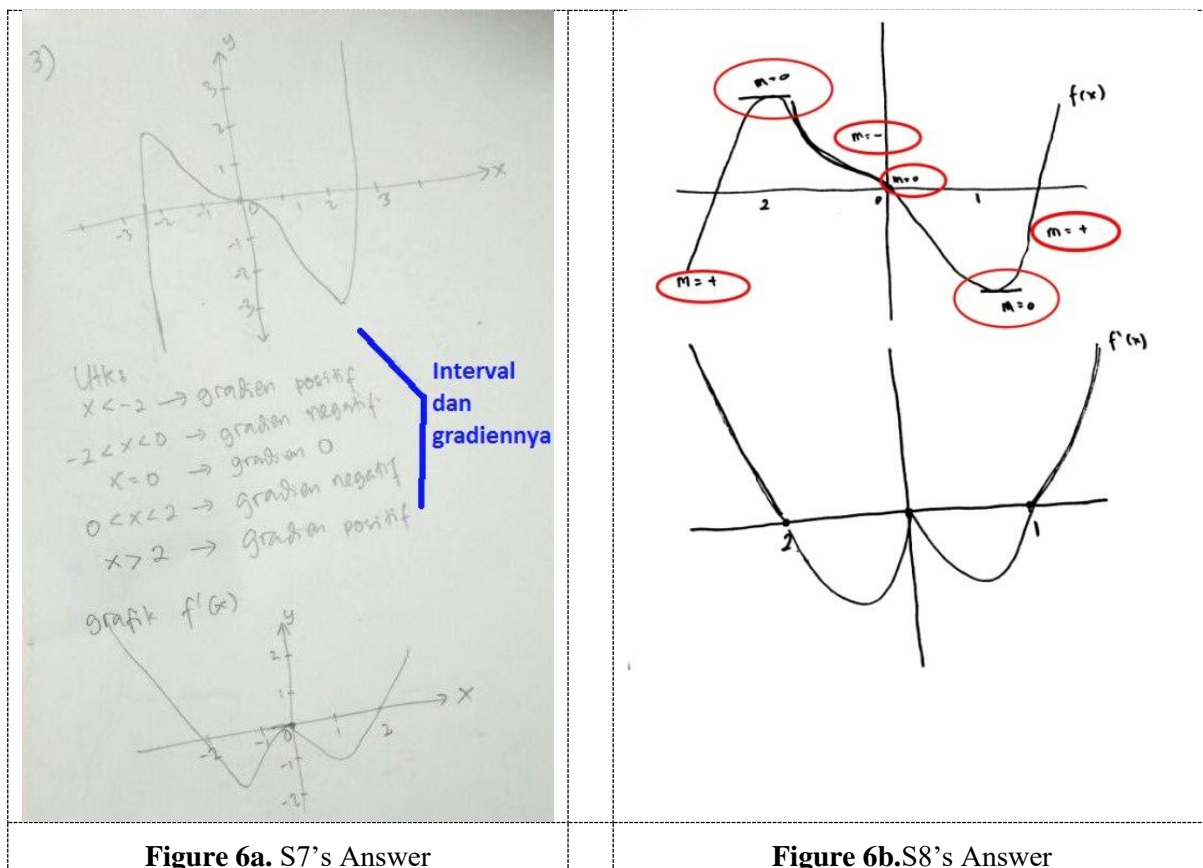
Figure 4b. Answer Type B

Besides, S6 first reads the graph based on the interval on the axis x S6 translates the graph given to the problem to a symbolic form of interval on the axis x and adjusted for its up and down function characteristics. S6 distinguishes three intervals ie, $x < -2$, $-2 < x < 2$ and $x > 2$ (Figure 5a). S6 interprets $x < -2$ as a function of rising to obtain $f'(x) < 0$, which resulted in sketsa graph at intervals are above the axis x to cut the point $x = -2$. S6 assumes that at intervals $-2 < x < 2$ is an ordinary straight line, there is no curve curve (Figure 5a red circle). Therefore, he interprets the graph at intervals $-2 < x < 2$ as a function down so it is obtained $f'(x) > 0$, which resulted in sketches of the graph at intervals below the axis x from the point $(-2, 0)$ to the point $(2, 0)$. S6 interprets $x > 2$ as a function of rising to obtain $f'(x) < 0$, which resulted in sketsa graph at intervals are above the axis x from the point $(2, 0)$. In the final answer S6 creates a graph of quadratic function curve (Fig. 5b) based on the calculations that it performs.



From S6's answer, it can be seen that the interval created does not look at the nature of the graph at the moment $x = -2$, $x = 0$, and $x = 2$. S6 only sees the point $(-2, 0)$ and $(2, 0)$ in determining the interval. S6 translates the graph (from problem) to the symbolic form (the function interval) but does not run completely right. S6 is less observant in determining the graphic properties at each point and interval, so not all intervals are translated into symbols. Like at the moment the graph in the dot $(0, 0)$ there is a curve so that the interval is formed $-2 < x < 0$ and $0 < x < 2$. Consequently the final answer S6 in determining the graph not exactly. However, S6 can know the meaning of the problem is to determine the graph $f'(x)$ of the graph representation $f(x)$ the given is not symbolic of $f'(x)$.

3.1.3. Answer Type C. Subjects of this type are only 2 students (S7 and S8). In this type of answer, S7 and S8 can solve the problem correctly. S7 first transfers the given graph into the interval form (Figure 6a). S6 forms 6 intervals from the graph $f(x)$ given to know the nature of the gradient. At the time of the graph $f(x)$ in position $x < -2$ and $x > 2$, S6 determines the function gradient is positive. So in the final answer formed a graph $f'(x)$ is above the axis x during the second interval. At the time of the graph $f(x)$ in position $-2 < x < 0$ and $0 < x < 2$, S2 determines the function gradient is negative. So the graph $f'(x)$ under the axis x during the interval. And at the time of the graph $f(x)$ in position $x = 0$, S6 specifies the function gradient equal to 0. So the graph $f'(x)$ are on point $(0, 0)$.



S8 solves the differential problem by first reading the vertex of the given graph. Based on interviews with S8 (transkip), the subject sees there are three peak points at the time $x = -2$, $x = 0$, and $x = 1$. S8 looks at the graph $f(x)$ as a mixture of quadratic function graphs and functions with the power of three. In deciding $f'(x)$, S8 uses the properties of the up and down function by looking at the value of the stationer.

S8: there is a curve with its peak point there are three ee ... minus two then zero and ee ... one
 ... of the curve being viewed inikan e combined with a quadratic function curve rank
 If looking for sketches gr a fik derived function I use the relationship e
 ... function up, when its function up means first positive derivative or greater than zero then when its function down later .. later the first derivative negative or smaller than zero.
 Well if from the ee ... the first graph I specify is what I understand is when $f(x)$ is lowered in the $f'(x)$ it produces a stationary value,
 ... the graph from minus 2 to the left is the function up, it means that if we see from poisisi gradien he gr a diennya positive so he goes up means that in the picture nant he eeee ... $f(x)$, $f'(x)$ it has a positive value ...

The stationary value becomes the reference S8 in determining the point of intersection $f'(x)$ on the axis x . In addition to viewing the stationary point, S8 also determines the gradient at each interval (Figure 6b) so as to determine the graph $f'(x)$. For the symbol " $m = +$ " signifies a positive gradient and its function rises so $f'(x)$ positive. For the symbol " $m = 0$ " signifies $f'(x)$ will cut the axis x at that 0 graded point. And for the symbol " $m = -$ " denotes the gradient $f(x)$ negative and its function down so $f'(x)$ negative.

4. Results

Based on several findings from this study, there are 3 types of subjects based on the characteristics of the subject's answer in solving the differential problem. In type A subject S1 fails to do translation (translation) between source representation (graph of polynomial function) to target representation (graph $f'(x)$). Subjects can not translate quadratic and polynomial functions. However, in a linear function, the subject succeeds in translating between representations. The subject knows the purpose of the question and can differentiate quadratic functions, but can not answer the problem correctly. This is probably because S1 has not mastered the concept of function.

In type B, the subject S2 – S6 did not solve the problem correctly. The four subjects are running the wrong completion procedure. S2, S3, and S4 do not understand the meaning of the problem because it can not translate the source representation (graph of the polynomial function ($f(x)$) to the target representation (graph $f'(x)$). The three subjects finish in a symbolic way as a final answer, rather than with a graph. S5 gives the final answer in the form of a graph $f'(x)$ according to what the question asked. The four subjects of type B can read the graphical characteristics of the given but not entirely polynomial functions. The subjects run improper procedures, resulting in incorrect results. It is possible that the subject of this type has not mastered the concept of function and the lack of multiple representation experiences obtained.

Besides, the subject S6 has been able to translate the source representation (graph of the polynomial function ($f(x)$) to the target representation (graph $f'(x)$). However, the lack of carefulness in examining graphic characteristics (source representation) results in improper target representation. While in type C, the subject S7 and S8 can solve the given differential problem. Both subjects give different representation notations. S7 solves the differential problem by translating the graph $f(x)$ to the symbolic form (interval) to obtain the graph $f'(x)$ as the target representation. While S8 only gives gradient notation (m) on the graph $f'(x)$ according to the nature of the function (function up, down and stationary point) to obtain the graph $f'(x)$.

The research data shows that subjects are able to translate graphical representations to symbolic. Two subjects who successfully solve differential problems, provide symbolic representations according to their understanding so that they can make precise graphical representations. The symbols S7 and S8 appear to help them represent the graph $f(x)$ to the graph $f'(x)$. S7 raises notations of interval and gradient statements. S8 raises the notation of the symbol " m " on the graph $f(x)$.

Some of subjects one run algorithms solving differential in a wrong way. This is because the subject mastery about the concept of function and subject experience about the various forms of function representation is very less. So in making a graph of the differential function, the subject made an error. These results are consistent with the research of several researchers, such as Norman & Prichard (1994), Baker et al., (2000), and Arslan (2010) that calculus students are difficulty in making graphs of differential function.

5. Conclusions

Based on the results of research and discussion, the representation of problem solving on the students of mathematics teacher candidate raises 4 types of settlement. In general, subjects are able to translate between representations. However, subjects who do not have sufficient mastery of the concept, can not solve the problem properly. In addition the subject raises some semiotic components in the form of symbolic notations and statements in accordance with the understanding of the subject itself to get a problem solving. Thus, the subject as a potential teacher of mathematics should enrich the experience of representation and balanced conceptualization so as to improve the mathematical ability that will be able to help students when they become teachers of mathematics.

In the future, students math teacher candidates need to deepen the understanding of various forms of representation on the concept of differential and master translation / translation between representations. In the study furthermore it is necessary to investigate in more detail how the process of representational translation is viewed from different perspectives. In addition, appropriate strategies for enhancing understanding of representation on the differential also need to be examined.

References

- [1] NCTM, 2000 *Principles and Standards for School Mathematics* Reston: VA: NCTM.
- [2] Hwang W and Chen N, 2007 Multiple Representation Skills and Creativity Effects on Mathematical Problem Solving using a Multimedia Whiteboard System Jian-Jie Dung Yi-Lun Yang **10** p. 191–212.
- [3] Pantziara M Gagatsis A and Elia I, 2009 Using diagrams as tools for the solution of non-routine mathematical problems p. 39–60.
- [4] Diezmann C M and English L D, 2001, Promoting the Use of Diagrams as Tools for Thinking, in *2001 National Council of Teachers of Mathematics Yearbook: The Role of Representation in School Mathematics*, A. A. Cuoco, Ed. (National Council of Teachers of Mathematics), p. 77–89.
- [5] Goldin G and Shteingold N, 2001, Systems of Representations and the Development of Mathematical Concepts, in *The Roles of Representation in School Mathematics*, p. 1–23.
- [6] Kalathil R R and Sherin M G, 2000 Role of students' representations in the mathematics classroom. in *In International Conference of the Learning Sciences: Facing the Challenges of Complex Realworld Settings* p. 27.
- [7] Pape S J and Tchoshanov M A, 2001 The role of representation (s) in developing mathematical understanding *Theory Pract.* **40**, 2 p. 118–127.
- [8] Goldin G and Kaput J, 1996 A joint perspective on the idea of representation in learning and doing mathematics *Theor. Math. Learn.* p. 397–430.
- [9] Post T R, 1988 *Teaching Mathematics in Grades K-8. Research Based Methods* 7 Wells Ave., Newton, MA 02159: Allyn and Bacon, Inc.
- [10] Janvier C E, 1987 *Problems of representation in the teaching and learning of mathematics* Hillsdale, NJ, US: Lawrence Erlbaum Associates, Inc.
- [11] Ainsworth S, 1999 The functions of multiple representations *Comput. Educ.* **33**, 2–3 p. 131–152.
- [12] Roth W-M and Bowen G M, 2001 Professionals Read Graphs : A Semiotic Analysis *J. Res. Math. Educ.* **32**, 2 p. 159–194.
- [13] Duval R, 2006, A cognitive analysis of problems of comprehension in a learning of mathematics, *Educational Studies in Mathematics*, **61**, 1–2. p. 103–131.
- [14] Bossé M J Adu-Gyamfi K and Chandler K, 2014 Students ' Differentiated Translation Processes *Int. J. Math. Teach. Learn.* 828 p. 1–28.
- [15] Duval R, 1999 Representation, Vision and Visualization: Cognitive Functions in Mathematical Thinking. Basic Issues for Learning *Proc. Twenty First Annu. Meet. North Am. Chapter Int. Group for Psychol. Math. Educ.* p. 3–26.
- [16] Duru A and Koklu O, 2011 International Journal of Mathematical Middle school students ' reading comprehension of mathematical texts and algebraic equations p. 37–41.
- [17] Kendal M and Stacey K, 2003 Tracing Learning of Three Representations with the Differentiation Competency Framework *Math. Educ. Res. J.* **15**, 1 p. 22–41.
- [18] Ross S C, 1996 *A modern course in calculus. Calculus: The Dynamics of Change* Washington, DC: Mathematical Association of America.
- [19] Sofronas K S DeFranco T C Vinsonhaler C Gorgievski N Schroeder L and Hamelin C, 2011 What does it mean for a student to understand the first-year calculus? Perspectives of 24 experts *J. Math. Behav.* **30**, 2 p. 131–148.
- [20] Arslan S, 2010 Traditional instruction of differential equations and conceptual learning *Teach. Math. its Appl. An Int. J. IMA* **29**, 2 p. 94–107.
- [21] Berry J S and Nyman M A, 2003 Promoting students' graphical understanding of the calculus *J. Math. Behav.* **22**, 4 p. 481–497.
- [22] García M Llinares S and Sánchez-Matamoros G, 2011 Characterizing thematized derivative schema by the underlying emergent structures *Int. J. Sci. Math. Educ.* **9**, 5 p. 1023–1045.
- [23] Norman F A and Prichard M K, 1994, Cognitive obstacles to the learning of calculus: a Kruketskiian perspective, in *MAA NOTES*, p. 65–78.
- [24] Baker B Cooley L and Trigueros M, 2000 National Council of Teachers of Mathematics *J. Res. Math. Educ.* **31**, 5 p. 557–578.

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